$B \to K$ Transition Form Factor with Tensor Current within the k_T Factorization Approach

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Abstract

In the paper, we apply the k_T factorization approach to deal with the $B \to K$ transition form factor with tensor current in the large recoil regions. Main uncertainties for the estimation are discussed and we obtain $F_T^{B\to K}(0) = 0.25 \pm 0.01 \pm 0.02$, where the first error is caused by the uncertainties from the pionic wave functions and the second is from that of the B-meson wave functions. This result is consistent with the light-cone sum rule results obtained in the literature.

PACS numbers: 12.38.Aw, 12.38.Lg, 13.20.He

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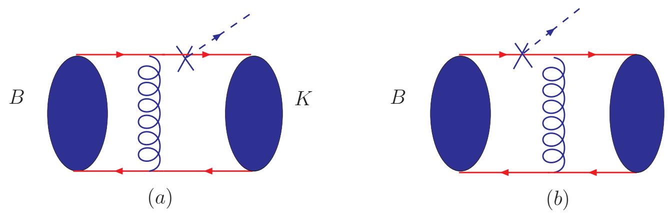

FIG. 1: Lowest order hard-scattering kernel for the $B \to K$ transition form factor, where the cross denotes an appropriate gamma matrix ($\sigma_{\mu\nu}$).

There is an increasing demand for more reliable QCD calculations of the heavy-to-light form factors, which plays a complementary role in determination of the fundamental parameters of the standard model and in developing the QCD theory. The $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ have been studied up to $\mathcal{O}(1/m_b^2)$ in the large recoil region within the k_T factorization approach [1], where the B-meson wave functions Ψ_B and $\bar{\Psi}_B$ that include the three-Fock states' contributions are adopted and the transverse momentum dependence for both the hard scattering part and the non-perturbative wave function, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reliable PQCD result. The rich flavor changing neutral current process $B \to K l^+ l^-$ have attracted people's attentions recently, since this decay provides potential testing grounds for the standard model at loop level and and is a hopeful channel to probe the new physics beyond the standard model, c.f. Ref.[2] and references therein. A better understanding of the rare semi-leptonic decay $B \to K l^+ l^-$ [3] needs a better understanding of its key component, i.e. the $B \to K$ transition form factor with tensor current $F_T^{B\to K}(q^2)$. So, in addition to $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$, it is also very interesting to study the properties of $F_T^{B\to K}(q^2)$, which is the purpose of the present letter.

The $B \to K$ transition form factor $F_T^{B\to K}(q^2)$ is defined as follows:

$$\langle K(P_K)|\bar{s}\sigma_{\mu\nu}q^\nu b|\bar{B}(P_B)\rangle = i\frac{F_T^{B\to K}(q^2)}{M_B + M_K}\left[q^2(P_B + P_K)_\mu - (M_B^2 - M_K^2)q_\mu\right], \tag{1}$$

where the momentum transfer $q = P_B - P_K$. The amplitude for the $B \to K$ transition form factor can be factorized into the convolution of the wave functions for the respective hadrons with the hard-scattering amplitude. In the large recoil regions, the $B \to K$ transition form factor is dominated by a single gluon exchange in the lowest order, whose Feynman diagram is shown in Fig.(1). In the hard scattering kernel, the transverse momentum in

the denominators are retained to regulate the endpoint singularity. The masses of the light quarks are neglected. The terms proportional to $\mathbf{k}_\perp^2$ or $\mathbf{l}_\perp^2$ in the numerator are dropped, which are power suppressed compared to other $\mathcal{O}(M_B^2)$ terms. Under these treatment, the Sudakov form factor from k_T resummation can be introduced into the PQCD factorization theorem without breaking the gauge invariance [4]. As for the $B \to K$ transition form factor $F_T^{B\to K}(q^2)$, it can be written in the transverse configuration b-space by properly including the Sudakov form factors and the threshold resummation effects:

$$\begin{aligned}
F_T^{B\to K}(q^2) \;=\; & \frac{\pi C_F}{N_c} f_K f_B M_B^2 \int d\xi dx \int b_B db_B\; b_K db_K\; \alpha_s(t) \times \exp[-S(x,\xi,b_K,b_B;t)] \\
& \times S_t(x) S_t(\xi) \Bigg\{ \Bigg[\Psi_K(x,b_K) \left(\Psi_B(\xi,b_B) - \bar{\Psi}_B(\xi,b_B)\right) + \frac{m_0^p}{M_B} \Psi_p(x,b_K) \cdot \\
& \left(\frac{1}{\eta} \bar{\Psi}_B(\xi,b_B) - x\Psi_B(\xi,b_B) \right) + \frac{m_0^p}{M_B} \frac{\Psi'_\sigma(x,b_K)}{6} \left(\frac{x\eta+2}{\eta} \Psi_B(\xi,b_B) \right. \\
& \left. - \frac{1}{\eta} \bar{\Psi}_B(\xi,b_B) \right) + \frac{m_0^p}{M_B} \frac{\Psi_\sigma(x,b_K)}{6} \Psi_B(\xi,b_B) \Bigg] h_1(x,\xi,b_K,b_B) - \frac{m_0^p}{M_B} \frac{\Psi_\sigma(x,b_K)}{6} \\
& [M_B \Delta(\xi,b_B)] h_2(x,\xi,b_K,b_B) + \Bigg[\Psi_K(x,b_K) \left(\frac{\Delta(\xi,b_B)}{M_B} - \xi \Psi_B(\xi,b_B) \right) + \\
& 2\frac{m_0^p}{M_B} \Psi_p(x,b_K) \left(\Psi_B(\xi,b_B) - \frac{\xi}{\eta} \bar{\Psi}_B(\xi,b_B) \right) \Bigg] h_1(\xi,x,b_B,b_K) \Bigg\},
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
h_1(x,\xi,b_K,b_B) \;=\; & K_0(\sqrt{\xi x \eta}\; M_B b_B) \Big[\theta(b_B - b_K) I_0(\sqrt{x\eta}\; M_B b_K) K_0(\sqrt{x\eta}\; M_B b_B) \\
& + \theta(b_K - b_B) I_0(\sqrt{x\eta}\; M_B b_B) K_0(\sqrt{x\eta}\; M_B b_K) \Big],
\end{aligned} \tag{3}$$

$$\begin{aligned}
h_2(x,\xi,b_K,b_B) \;=\; & \frac{b_B}{2\sqrt{\xi x y} M_B} K_1(\sqrt{\xi x \eta}\; M_B b_B) \Big[\theta(b_B - b_K) I_0(\sqrt{x\eta}\; M_B b_K) K_0(\sqrt{x\eta}\; M_B b_B) \\
& + \theta(b_K - b_B) I_0(\sqrt{x\eta}\; M_B b_B) K_0(\sqrt{x\eta}\; M_B b_K) \Big].
\end{aligned} \tag{4}$$

The functions I_i (K_i) are the modified Bessel functions of the first (second) kind with the *i-th* order. The angular integrations in the transverse plane have been performed. The factor $\exp(-S(x,\xi,b_K,b_B;t))$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude,

$$S(x,\xi,b_K,b_B;t) = \left[s(x,b_K,M_b) + s(\bar{x},b_K,M_b) + s(\xi,b_B,M_b) - \frac{1}{\beta_1} \ln \frac{\hat{t}}{\hat{b}_K} - \frac{1}{\beta_1} \ln \frac{\hat{t}}{\hat{b}_B} \right], \tag{5}$$

where $\hat{t} = \ln(t/\Lambda_{QCD})$, $\hat{b}_B = \ln(1/b_B\Lambda_{QCD})$, $\hat{b}_K = \ln(1/b_K\Lambda_{QCD})$ and $s(x,b,Q)$ is the Sudakov exponent factor, whose explicit form up to next-to-leading log approximation can

be found in Ref.[5]. $S_t(x)$ and $S_t(\xi)$ come from the threshold resummation effects and here we take a simple parametrization proposed in Refs.[4, 6],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c \ , \tag{6}$$

where the parameter c is determined around 0.3 for the present case. The hard scale t in $\alpha_s(t)$ and the Sudakov form factor might be varied for the different hard scattering parts and here we need two t_i[4, 7], whose values are chose as the largest scale of the virtualitiies of internal particles, i.e.

$$t_1 = \mathrm{MAX}\left(\sqrt{x\eta}M_B, 1/b_K, 1/b_B\right), \ t_2 = \mathrm{MAX}\left(\sqrt{\xi\eta}M_B, 1/b_K, 1/b_B\right). \tag{7}$$

The Fourier transformation for the transverse part of the wave function is defined as

$$\Psi(x,\mathbf{b}) = \int_{|\mathbf{k}|<1/b} d^2\mathbf{k}_\perp \exp\left(-i\mathbf{k}_\perp\cdot\mathbf{b}\right)\Psi(x,\mathbf{k}_\perp), \tag{8}$$

where Ψ stands for Ψ_K, Ψ_p, Ψ_σ, Ψ_B, $\bar{\Psi}_B$ and Δ, respectively. The upper edge of the integration $|\mathbf{k}_\perp| < 1/b$ is necessary to ensure that the wave function is soft enough [8].

In the numerical calculations, we use

$$\Lambda^{(n_f=4)}_{\overline{MS}} = 250MeV, \quad f_B = 190MeV, \quad M_B = 5.279GeV. \tag{9}$$

Further more, we need to know the non-perturbative wave functions for the B meson and kaon. Here we take the models as adopted in Ref.[1] to do our calculation, where only the kaon twist-2 wave function should be slightly changed to include the second Gegenbauer moment a_2^K's effect as suggested in Ref.[9], i.e.

$$\Psi_K(x,\mathbf{k}_\perp) = [1+B_K C_1^{3/2}(2x-1)+C_K C_2^{3/2}(2x-1)]\frac{A_K}{x(1-x)}\exp\left[-\beta_K^2\left(\frac{k_\perp^2+m_q^2}{x}+\frac{k_\perp^2+m_s^2}{1-x}\right)\right], \tag{10}$$

where $q=u,\ d$, $C_1^{3/2}(1-2x)$ is the Gegenbauer polynomial. The constitute quark masses are set to be: $m_q = 0.30$GeV and $m_s = 0.45$GeV. It can be found that the $SU_f(3)$ symmetry is broken by a non-zero B_K and by the mass difference between the s quark and u (or d) quark in the exponential factor. So the $SU_f(3)$ broken effects to the form factor are naturally included into our discussions. We will take $a_1^K(1GeV) = 0.05 \pm 0.02$ to determine the wave function Ψ_K. As for a_2^K, since it is still determined with large uncertainty and for convenience, we fix its value to be $a_2^K(1GeV) = 0.115$ [10]. The four parameters A_K, B_K, C_K

	this work	LCSR [14]	LCSR [10]
$F_T^{B\to K}(0)$	$0.25 \pm 0.01 \pm 0.02$	0.27 ± 0.04	0.321 ± 0.046

TABLE I: Simple comparison of $F_T^{B\to K}(q^2=0)$ calculated within the present adopted k_T factorization approach and the LCSR approach [10, 14].

and β_K can be determined by its first two Gegenbauer moments a_1^K and a_2^K, the constraint $\langle \mathbf{k}_\perp^2 \rangle_K^{1/2} \approx 0.350$GeV [11] and the normalization condition $\int_0^1 dx \int_{k_\perp^2<\mu_0^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_K(x,\mathbf{k}_\perp) = 1$. For example, we have $A_K(\mu_b) = 252.044 GeV^{-1}$, $B_K(\mu_b) = 0.09205$, $C_K(\mu_b) = 0.05250$ and $\beta_K = 0.8657 GeV^{-1}$ for the case of $a_1^K(1GeV) = 0.05$ and $a_2^K(1GeV) = 0.115$. As for the B meson wave functions, we adopt the simple model raised in Ref.[12] to do our discussion:

$$\Psi_B^+(\xi,b_B) = (16\pi^3)\frac{M_B^2\xi}{\omega_0^2}\exp\left(-\frac{M_B\xi}{\omega_0}\right)\Big(\Gamma[\delta]J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta]J_{1-\delta}[\kappa]\Big)\left(\frac{\kappa}{2}\right)^{1-\delta} \quad (11)$$

and

$$\Psi_B^-(\xi,b_B) = (16\pi^3)\frac{M_B}{\omega_0}\exp\left(-\frac{M_B\xi}{\omega_0}\right)\Big(\Gamma[\delta]J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta]J_{1-\delta}[\kappa]\Big)\left(\frac{\kappa}{2}\right)^{1-\delta}, \quad (12)$$

which satisfy the normalization $\int \frac{d\xi d^2\mathbf{k}_\perp}{16\pi^3}\Psi_B^\pm(\xi,\mathbf{k}_\perp) = 1$. $\omega_0 = 2\bar\Lambda/3$, $\bar\xi = \bar\Lambda/M_B$, $\kappa = \theta(2\bar\xi-\xi)\sqrt{\xi(2\bar\xi-\xi)}M_B b_B$. According to the definitions, we have $\Psi_B(\xi,b_B) = \Psi_B^+(\xi,b_B)$, $\bar\Psi_B(\xi,b_B) = \Psi_B^+(\xi,b_B) - \Psi_B^-(\xi,b_B)$ and $\Delta(\xi,b_B) = -M_B\int_0^\xi d\xi'\bar\Psi_B(\xi',b_B)$. The B-meson wave function Fock state expansion depends on two phenomenological parameters $\bar\Lambda$ and δ. We will take $\bar\Lambda \in [0.50, 0.55] GeV$ and $\delta \in [0.25, 0.30]$ to study their uncertainties to the form factor $F_T^{B\to K}(q^2)$, which is determined by comparing the PQCD results of $B\to\pi$ form factor with the QCD LCSR results and lattice QCD calculations [13].

By varying the undetermined parameters, such as a_1^K, $\bar\Lambda$ and δ, we compare our results of $F_T^{B\to K}(q^2=0)$ with those derived from the QCD light-cone sum rules in TAB.I. We obtain $F_T^{B\to K}(q^2=0) = 0.25\pm0.01\pm0.02$, where the center value is obtained by setting $a_1^K(1GeV) = 0.05$, $\bar\Lambda = 0.525 GeV$ and $\delta = 0.275$, and the first error comes from the uncertainty of a_1^K and the second comes from that of $\bar\Lambda$ and δ. Our result shows a good agreement with the LCSR result of Ref.[14], and both of which roughly agree with the result of Ref.[10] within theoretical errors. Further more, it can be found that the PQCD results can match with the LCSR results for small q^2 region, e.g. $q^2 < 10 GeV^2$. Then by combining the PQCD results with the LCSR results, we can obtain a consistent analysis of the form

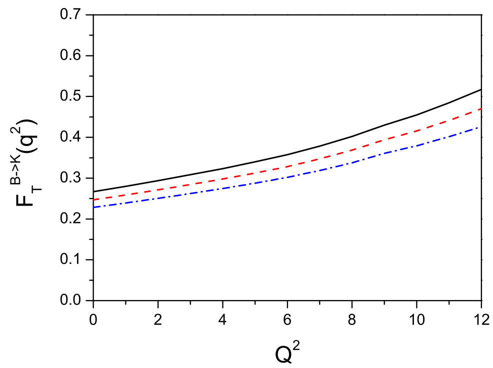

FIG. 2: PQCD results for the $B \to K$ form factors $F_T^{B\to K}(q^2)$ with fixed $\delta = 0.275$ and $a_1^K(1GeV) = 0.05$. The solid line, the dashed line and the dash-dot line are for the cases of $\bar{\Lambda} = 0.500GeV$, $0.525GeV$ and $0.550GeV$ respectively.

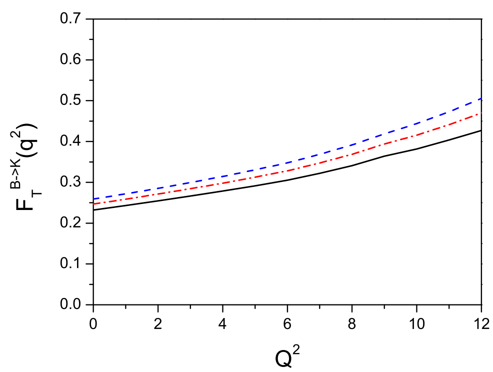

FIG. 3: PQCD results for the $B \to K$ form factors $F_T^{B\to K}(q^2)$ with $\bar{\Lambda} = 0.525GeV$ and $a_1^K(1GeV) = 0.05$. The solid line, the dash-dot line and the dashed line stand for the cases of $\delta = 0.25$, 0.275 and 0.30 respectively.

factor within the large and the intermediate energy regions [9, 13]. Inversely, if the PQCD approach must be consistent with the LCSR approach, then we can obtain some constraints to the undetermined parameters within both approaches.

It may be interesting to know how the undermined parameters, such as a_1^K, $\bar{\Lambda}$ and δ, affect the form factor $F_T^{B\to K}(q^2)$.

We first discuss the uncertainties of $F_T^{B\to K}(q^2)$ arise from the B-meson wave function, i.e. to discuss the uncertainties from $\bar{\Lambda}$ and δ. For such purpose, we fix the kaonic wave

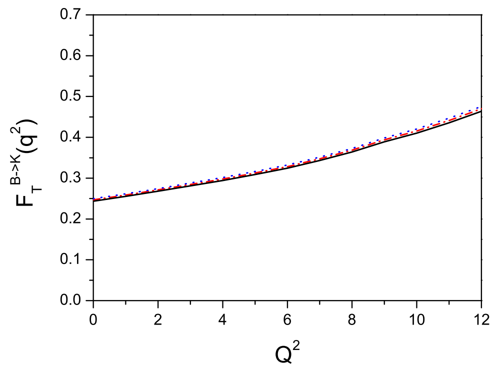

FIG. 4: PQCD results for the $B \to K$ form factors $F_T^{B\to K}(q^2)$ with $\bar{\Lambda} = 0.525GeV$ and $\delta = 0.275$. The solid line, the dash-dot line and the dotted line stand for $a_1^K(1GeV) = 0.03, 0.05$ and 0.07 respectively.

function by setting $a_1^K(1GeV) = 0.05$. The transition form factor $F_T^{B\to K}(q^2)$ with fixed $\delta = 0.275$ is shown in Fig.(2), where $\bar{\Lambda}$ varies within the region of $[0.50GeV, 0.550GeV]$. Fig.(2) shows that $F_T^{B\to K}(q^2)$ decreases with the increment of $\bar{\Lambda}$. While the transition from factor $F_T^{B\to K}(q^2)$ with fixed $\bar{\Lambda} = 0.525GeV$ is shown in Fig.(3), where δ varies within the region of $[0.25, 0.30]$. Fig.(3) shows that $F_T^{B\to K}(q^2)$ increases with the increment of δ. As a whole, it can be found that by varying $\bar{\Lambda} \in [0.50GeV, 0.550GeV]$ and $\delta \in [0.25, 0.30]$, then it will cause about 10% uncertainty to $F_T^{B\to K}(q^2)$.

Second, we discuss the properties of $F_T^{B\to K}(q^2)$ caused by the twist-2 wave function Ψ_K, ie. by the value of $a_1^K(1GeV)$. For this purpose, we fix the B-meson wave functions by setting $\bar{\Lambda} = 0.525GeV$ and $\delta = 0.275$. We show the $B \to K$ transition form factor $F_T^{B\to K}(q^2)$ in Fig.(4) with $a_1^K(1GeV) = 0.03$, 0.05 and 0.07 respectively. It is found that $F_T^{B\to K}(q^2)$ will slightly increase with the increment of $a_1^K(1GeV)$. And by varying $a_1^K(1GeV) \in [0.03, 0.07]$, it will cause about 1% uncertainty to $F_T^{B\to K}(q^2)$.

In summary: we have applied the k_T factorization approach to calculate the $B \to K$ transition form factor $F_T^{B\to K}(q^2)$ up to order $(1/m_b^2)$, where the transverse momentum dependence for the wave function, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reasonable result. By varying the undetermined parameters, such as a_1^K, $\bar{\Lambda}$ and δ, within the reasonable reasonable regions, we obtain $F_T^{B\to K}(0) = 0.25 \pm 0.01 \pm 0.02$. It shows that the k_T factorization can be applied

to calculate the form factors in the large recoil regions. Together with the newly developed PQCD results of $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ [1], one can achieve a full understanding of the these three $B \to K$ form factors in the large recoil regions. Furthermore, in combination of the LCSR results, one can know well the $B \to K$ form factor $F_T^{B\to K}(q^2)$ in the large and intermediate energy regions and then to derive a better understanding of the rare semi-leptonic decay $B \to K l^+ l^-$ [3].

Acknowledgement: This work was supported in part by the Natural Science Foundation of China (NSFC), by the Grant from Chongqing University and by the National Basic Research Programme of China under Grant NO. 2003CB716300.

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